Weźmy dowolne $z \in \mathbb{C}$ takie, że $|z|=1$.

Wówczas $\exists x \in \mathbb{R}: \; z = \cos x + i\sin x$

$$z^3 \stackrel{\text{de Moivre}}{=\!=\!=} \cos 3x + i\sin 3x \quad (1)$$

$$z^3 = (\cos x + i\sin x)^3 =$$

$$= \cos^3 x + 3\cos^2 x\, i\sin x \cancel{+} - 3\cos x\sin^2 x \cancel{+} - i\sin^3 x$$

$$= (\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x) \quad (2)$$

z (1) $\wedge$ (2):

$$\begin{cases} \cos 3x = \cos^3 x - 3\cos x\sin^2 x \\ \sin 3x = 3\cos^2 x\sin x - \sin^3 x \end{cases}$$

$\blacksquare$